

Q.7 — Sierpiński Square Carpet

Question: Stage 0 of the Sierpiński square carpet is a square sheet of paper. To construct Stage 1, each side of the square is trisected and the points of trisection of opposite sides are joined to obtain nine smaller squares. The centre square is removed and the 8 smaller squares are retained. The same process is repeated on the eight smaller shaded squares to obtain Stage 2 and so on.

(i) Number of red squares in Stages 0 to 3

Stage 0: 1 red square.

Stage 1: 8 red squares.

Stage 2: $8 \times 8 = 64$ red squares.

Stage 3: $64 \times 8 = 512$ red squares.

Therefore, the numbers of red squares are 1, 8, 64 and 512.

(ii) Number of red squares in Stages 4 and 5

The number of red squares at Stage n is $N_n = 8^n$.

Stage 4: $N_4 = 8^4 = 4096$.

Stage 5: $N_5 = 8^5 = 32768$.

(iii) Rule for the number of red squares

Explicit formula: $N_n = 8^n$, where $n = 0, 1, 2, \dots$

Recursive formula: $N_0 = 1$, and $N_n = 8N_{n-1}$ for $n \geq 1$.

(iv) Area of the red region

Let the area of the square at Stage 0 be 1 square unit.

At each stage, each square is divided into 9 equal squares and the central square is removed. Thus, 8 out of the 9 equal parts remain red.

Stage 1: $A_1 = 8/9$.

Stage 2: $A_2 = (8/9)(8/9) = (8/9)^2 = 64/81$.

Stage 3: $A_3 = (8/9)^3 = 512/729$.

Stage 4: $A_4 = (8/9)^4 = 4096/6561$.

Stage 5: $A_5 = (8/9)^5 = 32768/59049$.

Explicit and recursive formula for the area

Explicit formula: $A_n = (8/9)^n$, where $n = 0, 1, 2, \dots$

Recursive formula: $A_0 = 1$, and $A_n = (8/9)A_{n-1}$ for $n \geq 1$.

Behaviour of the area as the number of stages increases

Since $0 < 8/9 < 1$, the value of $(8/9)^n$ decreases towards 0 as n increases.

Therefore, the area of the red region approaches 0 square units as the number of stages tends to infinity.